1. Find $2 \times (2+3)$.

   (A) 8 (B) 9 (C) 10 (D) 11 (E) 30

   Possible Solution:
   $2 \cdot 5 = 10$.

2. On Monday, Lyndon receives a 80 on his daily math quiz. After being scolded by his parents, he works harder and gets an 83 on Tuesday. From Tuesday onward, his score improves by 3 points each day. What will be Lyndon's score that Friday?

   (A) 89 (B) 92 (C) 95 (D) 98 (E) 100

   Possible Solution:
   Tues, Wed, Thurs, Fri make 4 days after Monday, so his score improves by 12.

3. Given that $2x + 5 - 3x + 7 = 8$, what is the value of x?

   (A) -4 (B) -2 (C) 0 (D) 2 (E) 4

   Possible Solution:
   $12 - x = 8, x = 4$

4. Allen flips a fair two sided coin and rolls a fair 6 sided die with faces numbered 1 through 6. What is the probability that the coin lands on heads and he rolls a number that is a multiple of 5?

   (A) $\frac{1}{24}$ (B) $\frac{1}{12}$ (C) $\frac{1}{6}$ (D) $\frac{1}{4}$ (E) $\frac{1}{3}$

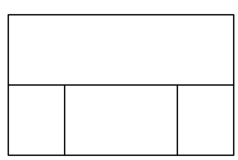
   Possible Solution:
   $\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$

5. How many distinct prime factors does the number 36 have?

   (A) 2 (B) 4 (C) 6 (D) 10 (E) 15

   Possible Solution:
   2 and 3 are the only prime factors.

6. How many rectangles are in the following figure?

   (A) 5 (B) 6 (C) 7 (D) 8 (E) 9

   Possible Solution:
   Considering only the bottom half, there are $3 + 2 + 1 = 6$ rectangles, then 1 more for the top and and one for the whole, giving 8.

7. In one peculiar family, the mother and the three children have exactly the same birthday. Currently, the mother is 37 years old while each of children are 9 years old. How old will the mother be when the sum of the ages of the three children equals her age?

   (A) 14 (B) 27 (C) 42 (D) 57 (E) 66

Possible Solution:
The difference between the parent and child age decreases by 2 each year, as each child grows 1 year older and there are three of them, while the mother gains 1 year by herself. The current difference is 10, so it will take 5 years. The mother's age will then be 42.

8. James randomly selects 4 distinct numbers between 3 and 10, inclusive. What is the probability that all 4 numbers are prime?

(A) 0 (B) $\frac{1}{28}$ (C) $\frac{1}{14}$ (D) $\frac{1}{7}$ (E) $\frac{1}{4}$

Possible Solution:
The only primes are 3, 5, 7, so it is impossible to select 4 of them.

9. Betsy is addicted to chocolate. Every day, she eats 2 chocolates at breakfast, 3 chocolates at lunch, 1 chocolate during her afternoon snack time, and 5 chocolates at dinner. If she buys a bag of 100 chocolates and begins eating it at breakfast the next morning, during which meal will she eat her last piece?

(A) breakfast (B) lunch (C) snack time (D) dinner (E) impossible to determine

Possible Solution:
Each day she consumes 11 chocolates, so in 9 days she will have eaten 99. Then there 1 will remain to be eaten for breakfast on the last day.

10. John defines the function $f(x) = (x-3)(x-9)+8$. What the value of $f(3)$?

(A) 0 (B) 3 (C) 8 (D) 9 (E) 12

Possible Solution:
$f(3) = 0 + 8 = 8$

11. Call a number N which satisfies exactly three of the four following conditions as *two-good*.

(I) N is divisible by 2
(II) N is divisible by 4
(III) N is divisible by 8
(IV) N is divisible by 16

How many integers between 1 and 100, inclusive, are *two-good*?

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

Possible Solution:
Note that if N is divisible by 16, then it would also be divisible by 8, 4, 2. Thus N cannot satisfy the 4th rule, meaning it must be divisible by 8 but not 16. There are 12 multiples of 8 under 100; half of them are divisible by 16, so the answer is 6.

12. Calculate the product $\frac{1}{3} \cdot \frac{2}{4} \cdot \frac{3}{5} \cdots \frac{18}{20} \cdot \frac{19}{21}$.

(A) $\frac{1}{210}$ (B) $\frac{1}{190}$ (C) $\frac{1}{21}$ (D) $\frac{1}{20}$ (E) $\frac{1}{10}$

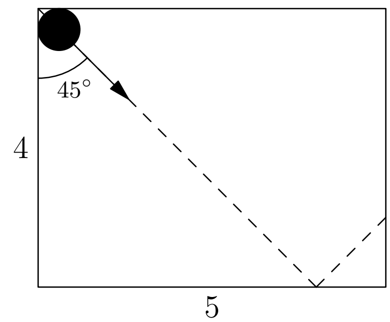
Possible Solution:
All but the first two numerators and last two denominators cancel, leaving $2/(20 \cdot 21) = 1/210$

13. What is the remainder when the sum $1 + 10 + 19 + 28 + \cdots + 91$ is divided by 9?

(A) 0 (B) 2 (C) 3 (D) 4 (E) 8

Possible Solution:
Each individual term leaves a remainder of 1 when divided by 9. There are 11 terms, so the sum would leave a remainder of 11, which is then a remainder of 2.

14. A cue ball is shot at a 45 degree angle from the upper right corner of a billiard table with dimensions 4 ft by 5 ft, as shown. How many times does the ball bounce before hitting another corner? Assume that when the ball bounces, its path is perfectly reflected. The final impact in the corner does not count as a bounce.



(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

Possible Solution:
Drawing out the collisions will show that 7 leads to a collision in the lower left. An alternative solution uses virtual images.

15. Alice, Bob, and Catherine decide to have a race. Alice runs at a speed of 3 feet per second, Bob runs at a speed of 5 feet per second. In the end, Bob finishes the same amount of time before Catherine as Catherine finishes before Alice. What was Catherine's speed?

(A) $\frac{15}{4}$ (B) 4 (C) $\frac{17}{4}$ (D) $\frac{9}{2}$ (E) impossible to determine

Possible Solution:
$\frac{d}{s} - \frac{d}{5} = \frac{d}{3} - \frac{d}{s} \implies s = 15/4$

16. For some constant b, the graph of $y = x^2 + b^2 + 2bx - b + 2$ has only one x intercept. What is the value of b?

(A) 1 (B) 2 (C) 4 (D) 8 (E) 10

Possible Solution:
Rewrite equation: $y = x^2 + 2bx + (b^2 - b + 2)$
Application of the discriminant: $\Delta = 4b^2 - 4(1)(b^2 - b + 2) = 0 \implies -4b + 8 = 0 \implies b = 2$

17. Jason has 5 pairs of socks, and each pair is a different color. He randomly selects 3 socks. What is the probability two of the three socks form a pair (i.e. two are the same color)?

(A) $\frac{1}{12}$ (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) $\frac{2}{3}$

Possible Solution:
There are 5 pairs to choose from for the pair, then 8 socks to choose for from the 3rd sock. This give a total of 40 sets of 3. Then there are $\binom{10}{3} = 120$ total combinations. $40/120 = 1/3$
An alternative solution is to consider each draw. The first scenario is that Jason chooses one of the socks in the pair, which can be any color, then the second sock of that pair (probability 1/9), then picks a third sock of any color (probability $8/8 = 1$). The second scenario is that Jason chooses one of the socks in the pair, which can be any color, then a second sock not in the pair (probability 8/9), then a third sock that is the second of the pair (1/8). The third scenario is that Jason first chooses a sock that is not in the pair

(probability 1), then a second sock that is in the pair (8/9, as it can't be the same color as the first), then the second sock of that pair (1/8). This gives a total of 1/3 as well.

18. A class of 10 children is divided into 5 pairs of partners. Each pair of partners sits next to each other and works together during class. One day, the teacher decides he wants to divide the class into two groups. In order to make sure the students work with new people, he makes sure not to put any student in the same group as his or her partner. How many different ways can he divide the class?

    (A) 2 (B) 5 (C) 10 (D) 16 (E) 32

    Possible Solution:
    The teacher can select either student from each pair, giving a total of 32. Then, each combination is counted twice, as choosing the alternate student from each pair produces the same division of groups (i.e. picking the first student each time is the same as picking the second student each time). Therefore the answer is $32/2 = 16$.

19. Let $S(n)$ denote the sum of digits of n (For example, $S(17) = 1 + 7 = 8$). If a positive two digit integer is randomly selected, what is the probability $S(S(n)) \geq 8$?

    (A) 0 (B) $\frac{1}{9}$ (C) $\frac{2}{9}$ (D) $\frac{11}{45}$ (E) $\frac{13}{45}$

    Possible Solution:
    Note that $S(n)$ is at most 18 because n can only be a 2 digit integer. Then $S(S(n))$ is at most 9. If $S(S(n)) = 8$, then $S(n) = 17, 8$. For the first case, $S(n) = 17 \implies n = 89, 98$. For $S(n) = 8$, $n = 17, 26, 35, 44, 53, 62, 71, 80$. This gives a total of 10 integers.
    If $S(S(n)) = 9$, then $S(n) = 18, 9$. If $S(n) = 18$, then $n = 99$. If $S(n) = 9$, $n = 18, 27, 36, 45, 54, 63, 72, 81, 90$. This gives a total of 10 integers.
    There are $99 - 10 + 1 = 90$ total two digit integers. $20/90 = 2/9$

20. Given $x^2 - 6xy + 9y^2 + |x - 3| = 0$, calculate $x + y$.

    (A) 1 (B) 3 (C) 4 (D) 16 (E) impossible to determine

    Possible Solution:
    This factors into $(x - 3y)^2 + |x - 3| = 0$. Note that neither of these terms can be less than zero, so both must be zero. Therefore $x = 3$, $y = 1$. Then the answer is 4.

21. The first 32 perfect squares, $1, 4, 9, 16, 25, \ldots, 961, 1024$ are combined together into one large number by appending their digits in succession, forming the number $N = 1491625\ldots9611024$. How many digits does N have?

    (A) 84 (B) 85 (C) 86 (D) 87 (E) 88

    Possible Solution:
    There are three 1-digit perfect squares $(1, 4, 9)$.
    $16 - 81 \implies 4^2 - 9^2$ are the 2-digit perfect squares. There are $9 - 4 + 1 = 6$ of them.
    $100 - 961 \implies 10^2 - 31^2$ are the 3-digit perfect squares. There are $31 - 10 + 1 = 22$ of them.
    1024 is the only 4-digit perfect square.
    The total number of digits is $3 \cdot 1 + 6 \cdot 2 + 22 \cdot 3 + 1 \cdot 4 = 85$.

22. Scientists perform an experiment on a colony of bacteria with an initial population of 32. The scientists expose the bacteria to alternating rounds of light and darkness. They first put the bacteria in a bright environment for one hour before placing it in a dark room for the second hour, and then repeating this process. Because they are vulnerable to light, the population of the bacteria will be halved in one hour of exposure to sunlight. However, in one hour of darkness, the population triples. How many hours will it take for the bacteria's population to exceed 150?

(A) between 4 and 5 (B) between 5 and 6 (C) between 6 and 7 (D) between 7 and 8 (E) between 8 and 9

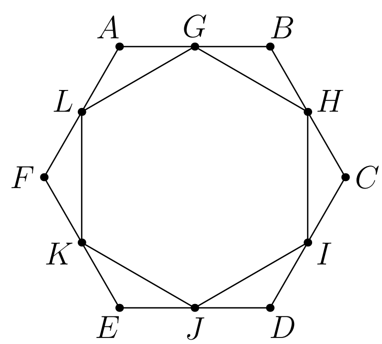
Possible Solution:
The initial population is 2^5. Over a two hour period of darkness then light, the population is multiplied by $3/2$. So after the first 2 hours, the population is $2^4 \cdot 3$. Then after the next two hours, the population is $2^3 \cdot 3^2$. Then after two more, the population is $2^2 \cdot 3^3$. Two more and the population is $2 \cdot 3^4$, which is greater than 150. Therefore, between the 7th and 8th hours, the population grows past 150.

23. How many ordered pairs of integers (x, y) satisfy $xy - 6y - 4x + 20 = 0$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 6

Possible Solution:
This factors into $(x-6)(y-4) = 4$. Because x, y are integers, each must be a factor of 4. The possible pairs of factors are $(1, 4)$, $(2, 2)$, $(4, 1)$, $(-1, -4)$, $(-2, -2)$, $(-4, -1)$. Each one produces a unique (x, y). Thus there are 6 pairs.

24. Regular hexagon $ABCDEF$ has area 1. Starting with edge AB and moving clockwise, a new point is drawn exactly one half of the way along each side of the hexagon. For example, on side AB, the new point, G, is drawn so $AG = \frac{1}{2}AB$. This forms hexagon $GHIJKL$, as shown. What is the area of this new hexagon?



(A) $\frac{3}{5}$ (B) $\frac{5}{7}$ (C) $\frac{3}{4}$ (D) $\frac{7}{9}$ (E) $\frac{4}{5}$

Possible Solution:
Calculating the ratio of the distances from the center to a vertex on each hexagon will give a way to calculate the ratio of the areas. Let the distance from the center to a vertex on the first hexagon be s. Then, through 30-60-90 triangles, the distance from the center to a vertex on the smaller hexagon will be $s\sqrt{3}/2$. Then the ratio of these lengths is $\sqrt{3}/2$. Then the ratio of areas is $(\sqrt{3}/2)^2 = 3/4$. Thus the area of the smaller hexagon is $3/4$. Alternatively, one may calculate the area of the missing triangles on each vertex. It is straightforward to find that each is exactly $1/4 \cdot 1/6 = 1/24$ of the area of the triangle, so all 6 together make up $1/4$, leaving $3/4$ remaining.

25. Each day John's mother sends him to the store with \$1 to buy widgets and gadgets, each of which cost a whole number of cents. On the first day John comes back with 4 widgets, 5 gadgets, and 35 cents in change. On the second day, John comes back with 5 widgets, 4 gadgets, and 39 cents in change. On the third day, John comes back with only c cents in change. He hands his mother the change, telling her that he had tripped coming home and broken all the widgets and gadgets. His mother, thinking for a moment, begins yelling at him for lying, as she noticed that there was no way he could have received exactly c cents in change given the price of widgets and gadgets. What is the sum of the digits of the least possible value of c?

(A) 10 (B) 13 (C) 15 (D) 18 (E) impossible to determine

Possible Solution:
$4w + 5g = 65$, $5w + 4g = 61$. Solving gives $w = 5, g = 9$. Applying Chicken McNugget theorem, the largest number not obtainable through any combination of 5s and 9s is $(5-1)(9-1)-1 = 31$. Therefore the least possible value of c is $100 - 31 = 69$, so the answer is 15.

Answer key

1. C
2. B
3. E
4. B
5. A
6. D
7. C
8. A
9. A
10. C
11. A
12. A
13. B
14. E
15. A
16. B
17. C
18. D
19. C
20. C
21. B
22. D
23. E
24. C
25. B