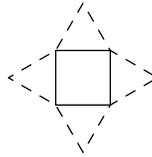
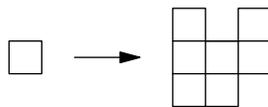




11. The figure below is used to fold into a pyramid, and consists of four equilateral triangles erected around a square with area nine. What is the length of the dashed path shown?



- (A) 18                      (B) 20                      (C) 21                      (D) 24                      (E) 27
12. Jo claims that any two triangles, both having a perimeter of four, are congruent. Jann claims that two circles, both having a circumference of  $4\pi$ , are congruent. Julia claims that two squares, both having a perimeter of four, are congruent. Which of these students are correct?
- (A) Jo                      (B) Jann                      (C) Julia                      (D) Jo, Julia                      (E) Jann, Julia
13. In a restaurant, a meal consists of one sandwich and one optional drink. In other words, a sandwich is necessary for a meal but a drink is not necessary. There are two types of sandwiches and two types of drinks. How many possible meals can be purchased?
- (A) 2                      (B) 4                      (C) 6                      (D) 12                      (E) 16
14. The notation  $[n]$  denotes the greatest integer less than or equal to  $n$ . Evaluate  $[2.1[-4.3]]$ .
- (A) -11                      (B) -10                      (C) -9                      (D) -8                      (E) -4
15. Which of the following answer choices is the closest approximation to
- $$\frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \cdots + \frac{1023}{1024} = \frac{2^2 - 1}{2^2} + \frac{2^3 - 1}{2^3} + \cdots + \frac{2^{10} - 1}{2^{10}}?$$
- (A)  $\frac{15}{2}$                       (B) 8                      (C)  $\frac{17}{2}$                       (D) 9                      (E)  $\frac{19}{2}$
16. A unit square block is attached to any place on the group of seven unit square blocks below such that it shares a side with at least one block.



What is the minimum possible perimeter of this new group of blocks?

- (A) 11                      (B) 12                      (C) 13                      (D) 14                      (E) 15
17. Cheryl rolls a fair dice twice. If the dice's six faces are numbered by 1, 2, 3, 4, 5, 6, what is the probability that the number on one of her rolls is a divisor of the number on the other roll?
- (A)  $\frac{2}{9}$                       (B)  $\frac{5}{18}$                       (C)  $\frac{4}{9}$                       (D)  $\frac{1}{2}$                       (E)  $\frac{11}{18}$

18. The buttons  $\{\times, +, \div\}$  on a calculator have their functions swapped. A button instead performs one of the other two functions; no two buttons have the same function. The calculator claims that  $2 + 3 \div 4 = 10$  and  $4 \times 2 \div 3 = 5$ . What does  $4 + 3 \times 2 \div 1$  equal on this calculator?
- (A) 4                      (B) 5                      (C) 7                      (D) 9                      (E) 10
19. Aprameya graphs the equation  $2x = y + 4$  on the coordinate plane. It turns out that there is a unique point with a positive integer coordinate and a negative integer coordinate lying on Aprameya's graph. What is the sum of the coordinates of this point?
- (A)  $-3$                       (B)  $-1$                       (C) 0                      (D) 1                      (E) 2
20. In a class of seven students, a poll is conducted. The poll asks
- (a) Were you born before 2021?  
(b) Were you born after 2019?  
(c) Were you born in 2020?

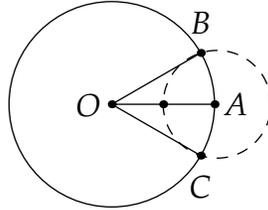
Five students respond "yes" to question (a), and four students respond "no" to question (b). If everyone truthfully answers all questions, how many students responded "yes" to (c)?

- (A) 1                      (B) 2                      (C) 3                      (D) 4                      (E) 5
21. You have a collection of \$20.21, consisting of pennies, nickels, and quarters. To reduce the collection's worth to  $k$  cents, you simultaneously replace all pennies with quarters and all quarters with pennies (all coins are replaced one time). What is the minimum possible  $k$ ?
- (A) 105                      (B) 120                      (C) 125                      (D) 505                      (E) 1011
22. Find the sum of all possible values of  $ab$ , given that  $(a, b)$  is a pair of real numbers satisfying

$$a + \frac{2}{b} = 9 \quad \text{and} \quad b + \frac{2}{a} = 1.$$

- (A)  $\frac{10}{9}$                       (B)  $\frac{3}{2}$                       (C) 3                      (D) 5                      (E) 9
23. Pikachu, Charmander, and Vulpix are three of the four equally-skilled players in a Pokemon bracket tournament. Because they are equally skilled, whenever any two of the players battle, they are equally likely to win. In the bracket tournament, the four players are randomly paired into two rounds, each round consisting of two players. The winners of the first two rounds then play each other in the final round. The winner of the final match ranks first; the loser of the final round ranks second; and the two losers of the previous rounds jointly rank third. What is the probability that Charmander plays Vulpix in a round, but ranks lower than Pikachu?
- (A)  $\frac{1}{24}$                       (B)  $\frac{1}{8}$                       (C)  $\frac{1}{3}$                       (D)  $\frac{3}{8}$                       (E)  $\frac{1}{2}$

24. On a circle  $O$  with radius  $\overline{OA}$ , points  $B$  and  $C$  are drawn such that  $\angle AOC = \angle BOA = 30^\circ$ , as shown. A second circle passing through  $B$ ,  $C$ , and the midpoint of  $\overline{OA}$  is drawn. The ratio of the radius of this new circle to the radius of circle  $O$  can be expressed in the form  $\frac{a\sqrt{3}-b}{c}$  where  $a$ ,  $b$ , and  $c$  are positive integers and  $c$  is as small as possible. What is  $a + b + c$ ?



Note: In the diagram,  $A$  is not necessarily the center of the second circle.

- (A) 10                      (B) 12                      (C) 15                      (D) 21                      (E) 27
25. Thelma writes a list of four digits consisting of 1, 3, 5, and 7, and each digit can appear one time, multiple times, or not at all. The list has a unique *mode*, or the number that appears the most. Thelma removes two numbers of that mode from the list; her list now has no unique mode! How many lists are possible? Suppose that all possible lists are unordered.
- (A) 18                      (B) 24                      (C) 30                      (D) 36                      (E) 48