2021 MIG Solutions

1. $20 - 2^1 = 20 - 2 = \mathbf{18}$.

2. Subtract 21 from both sides; $20x = 100$, and $x = \mathbf{5}$.

3. 20% of 10 is $\frac{20}{100} \cdot 10 = 2$, and 2 is 5% of $2 \cdot \frac{100}{5} = \mathbf{40}$.

4. Let there be x foxes. Then there are $x - 5$ rabbits and $x - 3$ pandas. $x - 3 > x - 5$, so there are more $(x - 3) - (x - 5) = \mathbf{2}$ more **pandas**.

5. Let k be the number Kermit erased. The sum of the numbers must be $1+2+3+4+5-k = 15-k$. So we must solve for k where $15 - k = 13$. The answer is $k = \mathbf{2}$.

6. $\mathbf{2^0 - 2 + 1 = 0}$ is the only even answer choice.

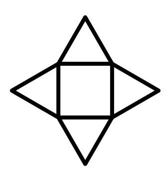
7. Denote the number of skittles as s. We want $|s - 5| = |s - 19|$, and the only positive solution s for this equation is **12**.

8. Note that a 15 by 17 rectangle has perimeter $2(15 + 17) = 64$. Suppose the square has side length s. We have $s^2 = 64$, or $s = 8$. The perimeter is $4s = \mathbf{32}$.

9. Let the teams be A, B, and C. If each team plays each other once, there would be three matches: $(A, B), (B, C), (A, C)$. However, each team plays twice, so the actual number of matches is $3 \cdot 2 = \mathbf{6}$.

10. Since k raisins distributed to 11 children leaves four raisins as leftover, $3k$ raisins distributed in the same manner would leave 12 raisins as leftover. But out of those 12 raisins, 11 can be distributed to the children evenly (each child gets one), so we have $12 - 11 = \mathbf{1}$ raisin left over.

11. The bolded line segments below all have the same length. Four of the bolded segments are sides of a square, which have length $\sqrt{9} = 3$. Our dotted path has eight of these segments, so its length is $8 \cdot 3 = \mathbf{24}$.



12. Two triangles with an equal perimeter do not necessarily have the same area, as their sides can differ (consider an equilateral and scalene triangle with the same perimeter). Two circles with the same perimeter must have the same radius, and thus the same area. Similarly, two squares with the same perimeter must have the same side lengths, and thus the same area. **Jann** and **Julia** are always correct.

13. Let the two types of sandwiches be A and B, while the drinks are C and D. A meal can consist of either sandwich A or B, while it can include no drink, drink C, or drink D. We have two cases for sandwiches and three cases for drinks; the answer is $2 \cdot 3 = \mathbf{6}$.

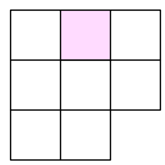
14. The tricky part is finding that $\lfloor -4.3 \rfloor = -5$, not -4. The inequality below shows why:

$$\cdots -5 < -4 < -3 < -2 < -1$$

Therefore, the expression evaluates to $\lfloor 2.1 \lfloor -4.3 \rfloor \rfloor = \lfloor 2.1 \cdot -5 \rfloor = \lfloor -10.5 \rfloor = \mathbf{-11}$.

15. Observe that $\frac{3}{4} = 1 - \frac{1}{2^2}, \frac{7}{8} = 1 - \frac{1}{2^3}$, and so on until $\frac{1023}{1024} = 1 - \frac{1}{2^{10}}$ We can then rewrite the entire sum as $1 - \frac{1}{2^2} + 1 - \frac{1}{2^3} + 1 - \frac{1}{2^4} + \cdots + 1 - \frac{1}{2^{10}} = 9 - (\frac{1}{2^2} + \cdots + \frac{1}{2^{10}})$. The number subtracted is arbitrarily close to $\frac{1}{2}$ (in fact, it is only $\frac{1}{2^{10}}$ off!), so our answer is $9 - \frac{1}{2} = \mathbf{\frac{17}{2}}$.

16. To minimize the perimeter, we want the unit square block to overlap the figure at as many sides as possible. This can be achieved in the diagram below (the attached block is colored):

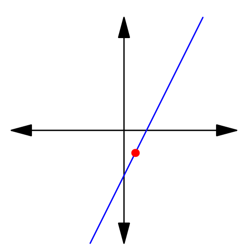


The perimeter of this figure is **12** units.

17. We must consider two cases: the two numbers are the same or the two numbers are different. There are 6 cases if the two rolled numbers are the same. However, we must double the count for two different numbers (consider switching the order of the rolls). In particular, the only cases to consider here are $(1,2), (1,3), (1,4), (1,5), (1,6), (2,4), (2,6), (3,6)$; so our total count is $6 + 8 \cdot 2 = 22$. The probability is then $\frac{22}{6 \cdot 6} = \mathbf{\frac{11}{18}}$.

18. A little bit of experimentation reveals that the $\times$ buttons performs as $\div$; the $+$ button performs as $\times$; the $\div$ button performs as $+$. This is because $2 \times 3 + 4 = 10$ and $4 \div 2 + 3 = 5$. Therefore, our answer is $4 \times 3 \div 2 + 1 = \mathbf{7}$.

19. The equation rearranges as $y = 2x - 4$. Graphing this equation gives the following:



As shown by the graph, the red point at $(1, -2)$ is the only point with a negative and positive integer coordinate, so our answer is $1 - 2 = \mathbf{-1}$.

20. Note that if four students responded "no" to question (b), then $7 - 4 = 3$ students responded yes to the same question. Therefore, five students were born before 2021 and three students were born after 2019. The overlap between these two groups of students is the number of students born in 2020. Since there are seven students, $5 + 3 - 7 = \mathbf{1}$ student was born in 2020.

21. To minimize k, we must maximize the number of quarters and minimize the number of pennies needed to form \$20.21. This can be achieved by eighty quarters, one penny, and four leftover nickels. The eighty quarters become pennies and the penny becomes a quarter. We must include four nickels to make up the remaining twenty cents. Therefore, $k = 80 \cdot 1 + 20 + 1 \cdot 25 = \mathbf{125}$.

22. Multiplying the two equations yields:

$$\left(a + \frac{2}{b}\right)\left(b + \frac{2}{a}\right) = ab + \frac{2}{a} \cdot a + \frac{2}{b} \cdot b + \frac{4}{ab} = 9,$$

Let $ab = k$. Then the equation becomes $k + \frac{4}{k} = 5$, which simplifies as $k^2 - 5k + 4 = 0$. By Vieta's Theorem, the sum of all k is $-(-5) = \mathbf{5}$.

23. First, we claim that Charmander must play Vulpix in the first round. If they do not, we can see that they will never have the opportunity to play each other. We are left with two scenarios: (i) Charmander wins against Vulpix, but then loses against Pikachu. This has a $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$ probability of occurring, since Pikachu must also win his first round. (ii) Vulpix wins against Charmander, and then either wins or loses against Pikachu. This has a $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ probability of occurring. Charmander has a $\frac{1}{3}$ probability of being paired with Vulpix in the first round, so our answer is $\frac{1}{3}(\frac{1}{4} + \frac{1}{8}) = \mathbf{\frac{1}{8}}$.

24. Note that $\triangle OBC$ is equilateral; suppose that it has a side length of 2. Let D be the intersection of BC and OA and M be the midpoint of OA. By some length chasing, we find that $DM = \sqrt{3} - 1, BC = 1$, and $BM = CM = \sqrt{1^2 + (\sqrt{3} - 1)^2} = \sqrt{5 - 2\sqrt{3}}$.

    We have obtained the side lengths of $\triangle BMC$, and now we can determine its circumradius:

$$\frac{CM \cdot BM \cdot BC}{4 \cdot [\text{area } \triangle ABC]} = \frac{5 - 2\sqrt{3}}{2(\sqrt{3} - 1)} = \frac{3\sqrt{3} - 1}{4}$$

    Dividing this by the radius of circle O yields $\frac{3\sqrt{3}-1}{8}$, so our desired answer is $3 + 1 + 8 = \mathbf{12}$.

25. The list can either have two of one number and two different numbers or three of one number and one other number. In the first case, there are $4 \cdot 3 \cdot 2 = 24$ possibilities. In the second case, there are $4 \cdot 3 = 12$ possibilities. In summary, we have $24 + 12 = \mathbf{36}$ total possibilities.