

1. $\frac{x-2}{3} = 6 \implies x = 20$
2. Because when grouped in fives, there are 3 left, and the number must be odd (if in pairs and there is 1 left, it must be odd), we are looking for a multiple of nine ending in 2. The smallest one that satisfies this is 72. Thus, our number is 73.
3. If all of the letters were unique, then the number of rearrangements would be $5!$. However, because there is one repeat, we must divide by $2!$. Thus, our answer is 60.
- 4.

$$\begin{aligned}
 (2x-1)^2 + (x+9)^2 &= (4x+1)^2 \\
 4x^2 - 4x + 1 + x^2 + 18x + 81 &= 16x^2 + 8x + 1 \\
 5x^2 + 14x + 82 &= 16x^2 + 8x + 1 \\
 11x^2 - 6x - 81 &= 0 \\
 (x-3)(11x+27) &= 0 \\
 x &= 3
 \end{aligned}$$

The legs have length 5 and 12. Therefore the area is 30.

5. We have that $y = x(x-8) * a$, where a is some constant. We can plug in $(4, -2)$ to see what a is. Doing this, we find that $a = \frac{1}{8}$. Thus, the linear term is $-8x * \frac{1}{8} = -1$.
6. The common difference is $-x+5$. We can find x in this way: $x+4-x+5 = 4x+1 \implies x=2$. From this, we can find y . The first 2 terms of the geometric sequence are 2, 6. Thus, the third term must be 18. $2y^2 = 18 \implies y=3$. Thus, our answer is $2+3=5$.
7. The space diagonal of a cube with side length s has length $s\sqrt{3}$. First, we must first the radius of the larger sphere.

$$\begin{aligned}
 \frac{4}{3}\pi r^3 &= 972\pi \\
 r &= 9
 \end{aligned}$$

Thus, the space diagonal must have length 18. $s\sqrt{3} = 18 \implies s = 6\sqrt{3}$. Thus, the radius of the smaller sphere must have length $3\sqrt{3}$. Thus, the volume of the smaller sphere must be $108\sqrt{3}\pi$. Our answer is then $972\pi - 108\sqrt{3}\pi$.

8.

$$\begin{aligned}
 (0.5)^{\frac{t}{60}} &= 0.1 \\
 \frac{t}{60} &= \log_{0.5} 0.1 \\
 t &= 60 \log_{0.5} 0.1 \\
 t &= 199.32
 \end{aligned}$$

9. The number of factors a number has can be found by finding its prime factorization, dropping each of the bases, adding 1 to each exponent, and multiplying all of the resulting numbers. For example, for $2^4 * 3^1$, we drop the bases. This gives us the numbers 4 and 1. We need to add 1 to each of these, to get 5 and 2. Then, we multiply these numbers to get 10. Thus, the number has 10 factors. For a number to have 6 factors, it either is in the form of a^5 or ab^2 . For the first case, only $2^5 = 32$ works. For the second case, we have $2 \times 3^2, 3 \times 2^2, 2 \times 5^2, 2 \times 7^2, 3 \times 5^2, 5 \times 2^2, 5 \times 3^2, 7 \times 2^2, 7 \times 3^2, 11 \times 2^2, 11 \times 3^2, 13 \times 2^2, 17 \times 2^2, 19 \times 2^2, 23 \times 2^2$. In the end we have 16 of these.
10. Because it is divisible by 5, the last digit is either 0 or 5. Because the last 4 digits are prime, it must be 5. Then, due to the fact that the three digit number made of the last three digits in order is divisible by 11, we can find that the last 3 digits must be 275. Thus, the fourth from last digit must be 3. Finally, using the calculator to guess and check will allow the first 4 digits to be found. Thus, the answer is 16403275.