1. ______________ An ant walks around on the coordinate plane. It moves from the origin to (3,4), then to (-9, 9), then back to the origin. How far did it walk? Express your answer as a decimal rounded to the nearest tenth.

Possible Solution:
$\sqrt{3^2+4^2}+\sqrt{12^2+5^2}+\sqrt{9^2+9^2}=30.7$

2. ______________ A cup with a volume of 8 fluid ounces is filled at the rate of 0.5 ounces per second. However, a hole at the bottom of the cup also drains it at the rate of 0.3 ounces per second. Once the cup is full, how many ounces of water will have drained out of the cup?

Possible Solution:
The net inflow is 0.2 ounces per second, meaning it will take $8/0.2 = 40$ seconds to fill the cup. In that time, $0.3 \cdot 40 = 12$ ounces will have drained

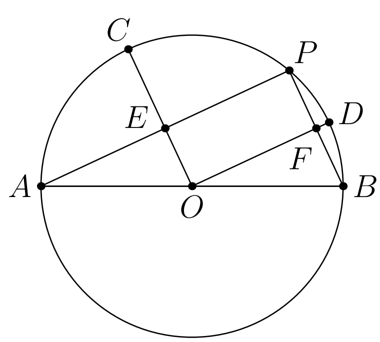
3. ______________ Calculate $1+2+3+4-5-6-7-8+9+\cdots-96+97+98+99+100$

Possible Solution:
If we pair up each group of 8, such as $1+2+3+4-5-6-7-8=-16$. There are 12 sets of these 8 numbers, then 4 additional numbers $(97+98+99+100)$ at the end. Thus the total sum is $-16\cdot 12+97+98+99+100=202$

4. ______________ AB is the diameter of circle O. A random point P is selected on O so that $AP = 4$ and $BP = 3$. Points C and D are drawn on circle O so that OC bisects AP and OD bisects BP. What is the circumradius of triangle ODC? Express your answer in simplest radical form.

Possible Solution:
Note that no matter how ODC is drawn, it is actually always a 45-45-90 triangle with right angle COD.

To show this, denote the intersection of OC and AP as E and the intersection of OD and PB as F. By Thales's theorem, angle APB will always be a right angle. Then $\angle OEP$ and $\angle OFP$ are also both right angles, as a radius that bisects a chord will always be perpendicular to it. Thus, in quadrilateral $OEPD$, three of the four angles are right angles. Therefore the fourth must also be a right angle, so $\angle EOF$ is a right angle. However, $\angle OEF = \angle COD$ and $OC = OD$, so triangle OCD is an isosceles right triangle no matter where P is located. Thus $\angle COD$ is 90 degrees.

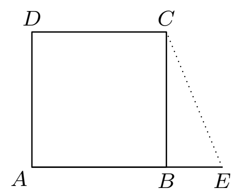


5. ______________ 3 builders are scheduled to build a house in 60 days. However, they suffer from a bout of procrastination and thus do nothing for the first 50 days. Panicked, they realize in order to build the house on time, they must hire more workers *and* work twice as fast as they would have before. If the new workers they hire also will work at the doubled

rate, how many new workers will the workers need to hire? Assume each builder works at the same rate as the others and they do not get in each others way.

Possible Solution:
Let the amount of work a builder working at his normal pace does in 1 day be a builder-day. Originally, the task required $3 \cdot 60$ builder-days. At the end, they still need to do this amount work, but working at double pace. This means that they can now accomplish 2 builder-days of work in 1 day. They have 10 days left. However, this means that there still needs to be $180/(2 \cdot 10) = 9$ builders present. Therefore they must hire $9 - 3 = 6$ more builders.

6. ______________ Square $ABCD$ has side length 4. Side AB is extended to point E so that AE has the same length as AC, as shown below. What is the length of EC? Express your answer as a decimal to the nearest hundredth.



Possible Solution:
The length of AC is $4\sqrt{2}$. Therefore the length of $BE = AE - AB = AC - AB = 4\sqrt{2} - 4 = 1.66$. Then, applying the Pythagorean Theorem to $\triangle EBC$, $EC = \sqrt{BE^2 + BC^2} = \sqrt{1.66^2 + 4^2} = 4.33$

7. ______________ How many positive integers less than or equal to 150 have exactly three distinct prime factors?

Possible Solution:
The only possible combinations of the 3 factors are $(2, 3, 5), (2, 5, 7), (2, 3, 7), (3, 5, 7)$.
Starting with the first, we have $2 \cdot 3 \cdot 5$. From this set, we can also make $2^2 \cdot 3 \cdot 5$, $2 \cdot 3^2 \cdot 5$, $2^3 \cdot 3 \cdot 5$, and $2 \cdot 3 \cdot 5^2$. Note that to find these other terms, we are basically multiplying $2 \cdot 3 \cdot 5$ by $2, 3, 4 = 2^2, 5$. Thus for this case there are 5 numbers.
For $2 \cdot 5 \cdot 7$, note that the best we can do is multiply by 2 to make $2^2 \cdot 5 \cdot 7 = 140$, as any further multiples would be greater than 150 or require us to use a 3. For this case there are just 2 numbers.
For $2 \cdot 3 \cdot 7 = 42$, we can multiply by 2 or 3, giving a total of 3 numbers.
For $3 \cdot 5 \cdot 7 = 105$, we cannot multiply by any other factors without exceeding 150, so there is just 1 number.
$5 + 2 + 3 + 1 = 11$

8. ______________ Greg plays a game in which he is given three random 1 digit numbers, each between 0 and 9, inclusive, with repeats allowed. He is to put these three numbers into any order. Exactly one ordering of the three numbers is correct, and if he guesses the correct ordering, he wins \$150. What are Greg's expected winnings for this game, given that he randomly guesses one valid ordering each time?

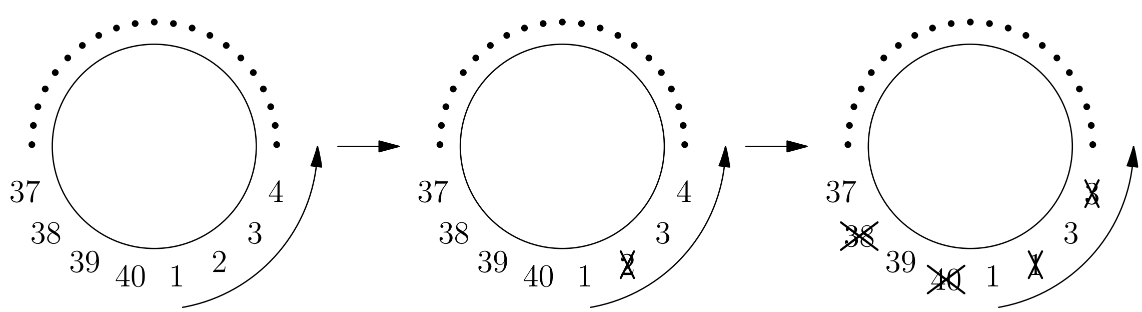
Possible Solution:
Let us first consider the case that all 3 numbers are different. There would be $10 \cdot 9\dot{8} = 720$ such sets of numbers. If all three numbers are different, then there exist $3! = 6$ permutations to choose from, so the probability of guessing one of these correct would be $1/6$

Now if two numbers are the same: We can choose any of the 10 numbers to be the repeated pair. Then there are 9 remaining possibilities for the third number. Then, for these 3 cards, there are 3 possible orderings (AAB, ABA, BAA), giving a total of $3 \cdot 10 \cdot 9 = 270$ possible groups of numbers. For these cards, there is a 1/3 chance of guessing the correct ordering, as established above.
Finally, if all three numbers are the same: There are only 10 of these possibilities. The chance of guessing the correct ordering for these is 1, as there is only one possible ordering.
We now calculate the weighted probability of guessing the correct ordering:
$\frac{720 \cdot 1/6 + 270 \cdot 3 + 10 \cdot 1}{720+270+10} = 0.22$. Then the expected winnings are $0.22 \cdot 150 = 33$. Notice that this also verifies that we counted every set, as $720 + 270 + 10 = 1000$, which we would expect from choosing three numbers, each ranging from 0 to 9.

9. ______________ Kevin develops a method for shuffling a stack of 10 cards numbered 1 through 10. He takes the top card off the unshuffled pile (which is in perfect order with 1 at the top and 10 at the bottom) and places it in what he calls the shuffled pile. Then, he flips a coin. If the coin is heads, he takes the card at the top of the unshuffled pile and places it at the top of the shuffled pile. If the coin comes up tails, he places the card at the at the bottom of the shuffled pile. He repeats this process for all the remaining cards. What is the probability that at the end of this shuffling, the top card is a prime number? Express your answer as a common fraction.

Possible Solution:
The primes are 2, 3, 5, 7.
In order for 2 to be at the top of the pile at the very end, every card following it must have been placed at the bottom of the shuffled pile AND 2 itself must have been placed at the top of the pile. This means that $2, 3, 4, 5, 6, 7, 8, 9, 10$, or 9 numbers needed to be placed on the top of the pile. That means the probability of this occuring was $(1/2)^9$.
By the same logic, for 3 to be on top of the pile at the end, it must have been placed at the top and every following card at the bottom. This gives a probability of $(1/2)^8$. Likewise, for 5 the probability is $(1/2)^6$ and for 7 the probability is $(1/2)^4$. Adding these together gives $(1/2)^9 + (1/2)^8 + (1/2)^6 + (1/2)^4 = 43/512$.

10. ______________ 40 people, numbered 1 through 40 counterclockwise, sit around a circular table. They begin playing a game. Each person is initially considered "alive". Starting with person 1, the first person eliminates the closest "alive" person to their right (so Person 1 eliminates Person 2). Then the next "alive" person, moving counterclockwise, eliminates the closest "alive" person to their right (so since Person 2 is eliminated, Person 3 eliminates Person 4). This process continues until there is only 1 "alive" person remaining. What is the number of the last "alive" person?



In the last step here, Person 39 eliminates Person 40. Next turn, Person 1 eliminates the closest person to his right, Person 3.

Possible Solution:
Note that when the number of people sitting around the table is a power of two, then the very first person (aka the person about to eliminate the next person at the time) will be the final winner. Thus, when there are 32 people left around the table, the person about to eliminate the next person will be the winner. So we must wait for 8 people to be eliminated first. Person 2, Person 4, 6, 8, 10, 12, 14, 16 will be the first 8 people eliminated. Then the next person to do the eliminating will be person 17, so they will be the final winner, as there will be 32 people remaining.

Answer key

1. 30.7
2. 12
3. 202
4. 5sqrt(2)/4
5. 6
6. 4.33
7. 11
8. 33
9. 43/512
10. 17