1. Calculate the numerical value of $1 \times 1 + 2 \times 2 - 2$.

   (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

   Possible Solution:
   $1 + 4 - 2 = 3$

2. A certain value of x satisfies $1 + x + 5 - 1 = 7$. What is this value of x?

   (A) 0 (B) 1 (C) 2 (D) 3 (E) impossible to determine

   Possible Solution:

$$\begin{aligned} 1 + x + 4 &= 7 \\ x + 5 &= 7 \\ x &= 2 \end{aligned}$$

3. What is the positive difference between the largest possible two-digit integer and the smallest possible three-digit integer?

   (A) 1 (B) 2 (C) 3 (D) 5 (E) 9

   Possible Solution:
   The largest possible two-digit integer is 99. THe smallest possible three digit integer is 100. Their difference is 1.

4. If you were to randomly select an answer to this question, what is the probability it would be correct?

   (A) 0% (B) 20% (C) 40% (D) 80% (E) 100%

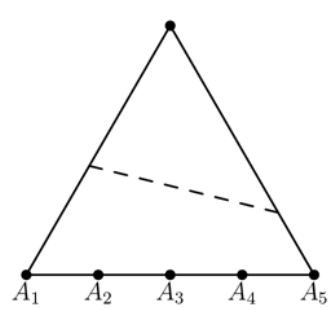
   Possible Solution:
   There are five answer choices, one of which is correct. Therefore the probability is $1/5 = 20\%$. We see that 20% is an available answer choice, so it must be the answer.

5. What is the side length, in meters, of a square with area 49 m^2?

   (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

   Possible Solution:
   The area of a square of side length x is x^2. Therefore $x^2 = 49 \implies x = 7$, since x must be positive.

6. The top vertex of this equilateral triangle is folded over the shown dashed line. Which of the 5 points will the vertex lie closest to after this fold?



(A) A_1 (B) A_2 (C) A_3 (D) A_4 (E) A_5

Possible Solution:
The dashed line is slanted, so the point must lie to the left of A_3. Eyeballing, it will definitely lie closer to A_2 than A_3.

7. John's digital clock is broken. It scrambles the digits of the time and displays them in a random order. For example, if the current time is 4 : 21, it could display 4 : 12, 2 : 14, or any other reordering of 4, 1, and 2. If his clock reads 6 : 71 one morning, how many possibilities are there for the correct time?

(A) 0 (B) 1 (C) 2 (D) 4 (E) 6

Possible Solution:
The possible times are 6 : 17 and 7 : 16. The time cannot start with 1, since that would make the minutes start with either 6 or 7, which is impossible.

8. $(1+\sqrt{3})^2$ may be written as $a+b\sqrt{3}$ for certain integers a and b. What is $a+b$?

(A) 1 (B) 2 (C) 4 (D) 6 (E) 7

Possible Solution:
$(x+y)^2 = x^2+2xy+y^2$. Therefore, $(1+\sqrt{3})^2 = 1+2\cdot 1\cdot\sqrt{3}+(\sqrt{3})^2 = 1+2\sqrt{3}+3 = 4+2\sqrt{3}$. Therefore $a=4$ and $b=2$ so $a+b=6$.

9. Lily has an unfair coin that has $\frac{2}{3}$ probability of showing heads and $\frac{1}{3}$ probability of showing tails. She flips the coin twice. What is the probability that the first flip is heads while the second is tails?

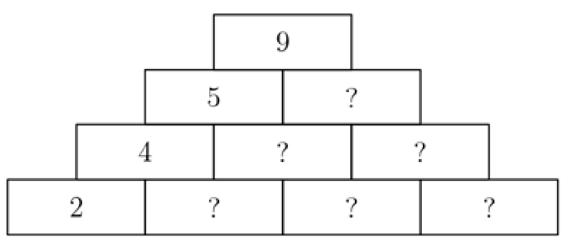
(A) 0 (B) 1/9 (C) 2/9 (D) 4/9 (E) 1

Possible Solution:
$\frac{2}{3}\cdot\frac{1}{3}=\frac{2}{9}$

10. In the diagram below, for each row except the bottom row, the number in each cell is determined by the sum of the two numbers beneath it. Find the sum of all cells denoted with a question mark.

(A) 6 (B) 8 (C) 12 (D) 13 (E) 14

Possible Solution:



Starting from the top, and listing left to right, the missing numbers are 4, 1, 3, 2, -1, 4. The sum of these is 13.

11. The numbers $1,2,3,4,5,6$ are placed onto the following six spots such that the average of the leftmost two spots, middle two spots, and rightmost two spots are all equal. What is the difference between the largest and smallest possibilities of the number on the shaded spot shown below?

○ ① ● ○ ○ ②

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Possible Solution:
If the average of each of the pairs is equal, then their sums must be equal. The sum of all 6 numbers is 21, so each sum must be 7. Therefore the leftmost circle is 6, the 5th circle is 5, and the two middle circles are 3 and 4 in any order. Therefore the largest possibility is 4 and the smallest is 3, so the difference is 1.

12. Jane's mother bakes cookies for Jane to share with her 6 friends. When the cookies are evenly divided among the 7 children (Jane and her 6 friends), there is one cookie left over. Given that each child receives at least 1 cookie, and Jane's mother baked less than 100 cookies, how many different numbers of cookies could Jane's mother have baked? For example, she could have baked 15 cookies, because each child receives 2 cookies, with 1 left over.

(A) 9 (B) 11 (C) 14 (D) 15 (E) 17

Possible Solution:
The number of cookies Jane's mother must have baked must be 1 more than a multiple of 7. These numbers are $8, 15, \ldots, 99$. There are $(99 - 8)/7 + 1 = 14$ such numbers.

13. For how many real values of x is the equation $(x^2 - 7)^3 = 0$ true?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Possible Solution:
$x = \pm\sqrt{7}$

14. Given that x satisfies $2^{4x} \cdot 4^{2x} \cdot 8^{4x} = 16^5$, find the value of x.

(A) 1 (B) 2 (C) 4 (D) 5 (E) 10

Possible Solution:

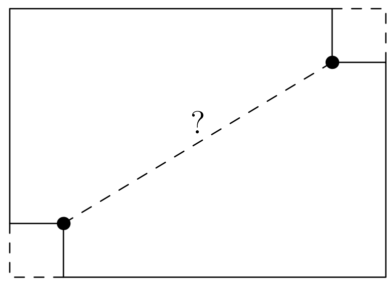
$2^{4x} \cdot 4^{2x} \cdot 8^{4x} = 2^{4x} \cdot (2^2)^{4x} \cdot (2^3)^{4x} = 2^{4x} \cdot 2^{4x} \cdot 2^{12x} = 2^{4x+4x+12x} = 2^{20x}$
$16^5 = (2^4)^5 = 2^{20}$
$2^{20x} = 2^{20}$, so $x = 1$

15. In the city of Urbextorto, the sales tax is 25%. A certain clothing store in the city is currently giving an n% discount on all its items, and n is special in that, after both the sales tax and discount are applied, a \$20 shirt ends up costing \$20. Find the value of n.

(A) 5 (B) 10 (C) 20 (D) 25 (E) 50

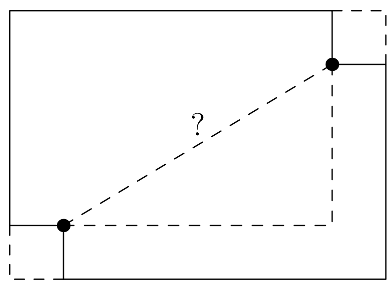
Possible Solution:
$20 \cdot (1 - n/100) \cdot 1.25 = 0 \implies n = 20$

16. Two 1 inch by 1 inch squares are cut out from opposite corners of a 7 inch by 5 inch piece of paper to form an octagon. What is the distance, in inches, between the two dotted points, both of which lie on corners of the octagon?



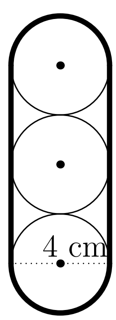
(A) 5 (B) $\sqrt{34}$ (C) $5\sqrt{2}$ (D) 8 (E) $\sqrt{74}$

Possible Solution:



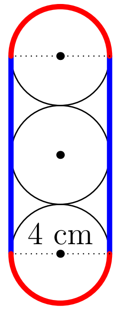
The right triangle shown has width $7 - 2 = 5$ and height $5 - 2 = 3$, so the hypotenuse has length $\sqrt{5^2 + 3^2} = \sqrt{34}$

17. A rubber band of negligible thickness encloses three pegs that lie in a perfect line, as shown. Each peg has a diameter of 4 cm, as shown. What is the length of the rubber band used, in centimeters? All pegs shown are congruent circles.



(A) 8 (B) $8 + 4\pi$ (C) $16 + 4\pi$ (D) $16 + 8\pi$ (E) 16π

Possible Solution:



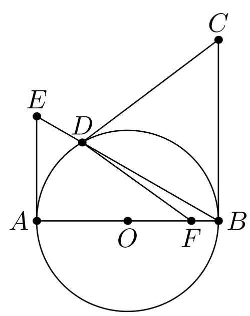
Blue segments have length 8. The red arcs each have length 2π. Therefore the total length is $16 + 4\pi$

18. When 171 is written as the sum of 19 consecutive integers, the median of those numbers is M. When 171 is written as the sum of 18 consecutive integers, the median of those numbers is N. Find $|M - N|$.

(A) -1 (B) -0.5 (C) 0 (D) 0.5 (E) 1

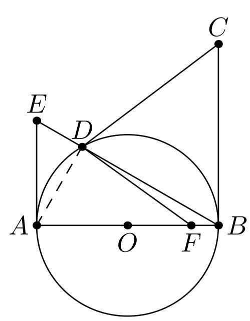
Possible Solution:
$171 = 19 \cdot 9$. The median of any arithmetic sequence is equal to its average. For the 19 integers, the average is 9. For the 18 integers, the average is $19/2$. The difference is therefore 0.5

19. In the diagram below, AB is a diameter of circle O. Point C is drawn such that $\overline{BC}$ is tangent to circle O, and $AB = BC$. A point F is selected on line AB and a point D is selected on circle O such that $\angle CDF = 90°$. Line $\overline{BD}$ is then extended to point E such that AE is tangent to circle O. Given that $AE = 5$, calculate the length of $\overline{AF}$. (Diagram not to scale)



(A) $\dfrac{9}{2}$ (B) 5 (C) $3\sqrt{3}$ (D) 7 (E) impossible to determine

Possible Solution:



Note that $\triangle ADB$ is a right triangle.
$\angle ADF = \angle ADB - \angle BDF = 90 - \angle BDF$. Also, $\angle BDC = \angle CDF - \angle BDF = 90 - \angle BDF$.

Therefore $\angle ADF = \angle BDC$.
$\angle DAB = 90 - \angle DBA$ and $\angle DBC = 90 - \angle DBA$ as well, so $\angle DAB = \angle DBC$.
Therefore $\triangle DAF \sim \triangle CDB$.

Since $\triangle EAB$ and $\triangle ADB$ are both right triangles and share $\angle ABD = \angle EBA$, they are similar as well.

Note that both of these pairs of similar triangles contain sides AD and DB.

Using $\triangle DAF \sim \triangle CDB$, $\frac{AD}{DB} = \frac{AF}{BC}$.

Using $\triangle EAB \sim \triangle ADB$, $\frac{AD}{DB} = \frac{AE}{AB}$.

Therefore $\frac{AF}{BC} = \frac{AE}{AB}$. $BC = AB \implies AE = AF = 5$.

20. John can purchase pieces of gum in packs of 4, 14, and 20 pieces. Given that he purchases at least one of each kind of pack, what is the positive difference between the greatest and least number of packs he can purchase to end up with exactly 86 pieces of gum?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

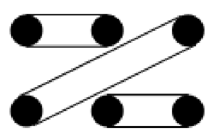
Possible Solution:
John must purchase at least 1 of each pack, which makes for $4+14+20 = 38$ pieces of gum. He still needs to purchase $86 - 38 = 48$ pieces. The least amount needed to make 48 pieces is 3 $(20 + 14 + 14 = 48)$. The most needed is $48/4 = 12$. Therefore the difference is 9.

21. Consider the following 2×3 arrangement of pegs on a board. Jane places three rubber bands on the pegs on the board such that the following conditions are satisfied:

(I) No two rubber bands cross each other.

(II) Each peg has a rubber band wrapped around it

How many distinct arrangements could Jane create exist? One acceptable arrangement is shown below.



(A) 2 (B) 3 (C) 5 (D) 6 (E) 8

Possible Solution:
Number the pegs from left to right, as 1, 2, 3 in the top row, and 4, 5, 6 in the bottom row. The shown arrangement is then 3-4, 1-2, 5-6. The other possibilities are as follows:

1-4, 2-5, 3-6
1-2, 4-5, 3-6
2-3, 5-6, 1-2
1-6, 2-3, 4-5

This gives 5 total arrangements.

22. Jane's uncle gives her a "4-balance." The 4-balance acts like a normal balance scale, but it compares four masses instead of two, tilting towards the weight that is heaviest (if all four are equal, it stays balanced). He then gives her 25 coins, one of which is a counterfeit heavier than the rest. What is the minimum number of uses of the 4-balance needed to ensure she identifies the counterfeit?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Possible Solution:
Divide the 25 coins into groups of five. Weigh four of these groups of four. If the counterfeit is in any

of these four groups, then that group will tilt downward. If it is not in any of those four groups, then the remaining 5 coins contains the counterfeit. Either way, we narrow the search to a group of five.

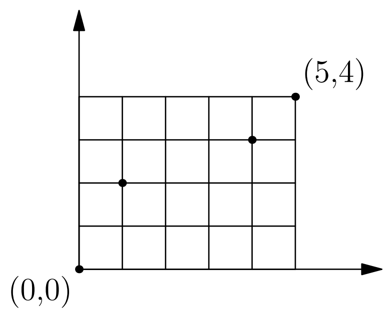
Now weigh four of these five. If the counterfeit is one of these four, it will tilt downwards. If none of the four coins is the counterfeit, then the final coin must be the counterfeit.

23. There exists a positive integer b such that the base-10 fraction $\frac{59}{48}$ can be expressed as $1.\overline{14}_b$ (or $1.141414..._b$), a value in base b. Find b.

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

Possible Solution:
$1.\overline{14}_b = 1 + b^{-1} + 4b^{-2} + b^{-3} + 4b^{-4} + \cdots = 1 + \frac{b^{-1}+4b^{-2}}{1-b^{-1}} = 1 + \frac{b+4}{b^2-1} = \frac{59}{48}$. Noting that $49 - 1 = 48$, we try $b = 7$, which works.

24. A leprechaun wishes to travel from the origin to a pot of gold located at the coordinate point (5,4). If she can only move upwards and rightwards along the unit grid, must pass a checkpoint at (1,2), and must avoid an evil thief at (4,3), how many distinct paths can she take?



(A) 7 (B) 15 (C) 21 (D) 45 (E) 126

Possible Solution:
To move to the checkpoint, the leprechaun must move 1 block right and 2 blocks up. This gives a total of $\binom{3}{1} = 3$ total paths.

Moving from the checkpoint to the pot of gold, ignoring the thief, requires 4 moves right and two moves up. This gives a total of $\binom{6}{2} = 15$ moves.

The number of paths from the checkpoint to the pot of gold that pass through the thief can be calculated as follows:

Moving from the checkpoint to the thief requires 3 moves right, 1 move up, for a total of $\binom{4}{1} = 4$ paths. Moving from the thief to the pot of gold requires 1 move up and 1 move right, for a total of 2 paths. Therefore 8 paths from the checkpoint to the pot of gold pass through the thief.

Thus, only $15 - 8 = 7$ paths from the checkpoint to the pot of gold do not pass through the thief. Thus, the answer is $3 \times 7 = 21$.

25. A number N is defined as follows:

$$N = 2 + 22 + 202 + 2002 + 20002 + \cdots + 2\overbrace{00\ldots000}^{19\text{ 0's}}2$$

When the value of N is simplified, what is the sum of its digits?

(A) 42 (B) 44 (C) 46 (D) 50 (E) 52

Possible Solution:
There are 21 terms in this sequence (2 with no zeroes, 19 with zeroes).

Adding together the units digits of all these terms is therefore equal to $21 \times 2 = 42$. Therefore the units digit of N is 2, and we carry 4 into the tens column. Note that the tens column only contains one 2, so $2 + 4 = 6$. All of the rest of the digits are then 2, meaning $N = 2 \ldots 262$. The largest term in the added terms has $19 + 2 = 21$ digits, so N must have 21 digits, so there are $21 - 2 = 19$ 2's which precede the 6. Thus, the answer is $19 \times 2 + 6 + 2 = 38 + 8 = 46$.

On the other hand, note that the sum of a numbers digits is congruent to the number itself modulo 9. Thus, if we take $N \equiv 2 + 4 + 4 + \cdots 4 \equiv 2 + 4 \times 20 \equiv 82 \equiv 1 \mod 9$. The only answer that is $1 \mod 9$ is 46.

Answer key

1. B
2. C
3. A
4. B
5. E
6. B
7. C
8. D
9. C
10. D
11. A
12. C
13. C
14. A
15. C
16. B
17. C
18. D
19. B
20. E
21. C
22. B
23. C
24. C
25. C