1. We have $4^0 = 1$, $3^1 = 3$, $2^2 = 2 \cdot 2 = 4$, and $1^3 = 1 \cdot 1 \cdot 1 = 1$. The desired quantity is $1 - 3 - 4 - 1 = -7$, so the answer is -7 **(B)**.

2. Because the machine makes coins at a constant rate, the machine will make one coin in $\frac{6}{4} = 1.5$ seconds. Thus, the machine makes 22 coins in $22 \cdot 1.5 = 33$ coins. The answer is 33 **(B)**.

3. After the change, Jar A will have $10 \cdot 2 = 20$ beans, and Jar B will have $10 \cdot \frac{1}{2} = 5$ beans. Then, there will be $20 + 5 = 25$ beans in total, corresponding to choice 25 **(C)**

4. We have $\sqrt{a^3b^2c} = \sqrt{(a^2 \cdot a)(b^2)(c)} = \sqrt{a^2b^2} \cdot \sqrt{ac} = ab\sqrt{ac}$ **(A)**.

5. Jamie will add 4 to 3 and subtract 2 from 1 before she multiplies the numbers, so her expression will simplify to 7×-1, which is equal to -7 **(A)**.

6. To minimize the amount of unit squares in the overlap of the two 3×3 squares, we must separate them as much as possible. This occurs if one grid is located in the top left corner while the other is in the bottom right corner. Then, they will only share the center square of the large grid. The answer is 1 **(B)**.

7. Split the line segment in the diagram along the vertical line between them. The left part of the line segment is half the length of the leftmost rectangle, while the right part of the line segment is half the length of the central rectangle.

   Thus the combined lengths of the first two rectangles is twice the length of the line segment, or $4 \cdot 2 = 8$. The last rectangle, then, must have length $12 - 8 = 4$ **(D)**.

8. The prime numbers from 1 to 10 are 2, 3, 5, and 7.

   The first time, the numbers 1, 4, 6, and 8 are erased, leaving the list $2, 3, 5, 7, 9, 10$.

   The second time, we erase the number 9, leaving 5 **(D)** numbers still in the list.

9. Consider the two inequalities $\frac{3}{2} < \frac{9}{x}$ and $\frac{9}{x} < \frac{7}{3}$. Since x is positive, we are allowed to cross-multiply the denominators. This yields the solutions $x < 6$ and $x > \frac{27}{7}$. The integer values of x satisfying both inequalities are $x = 4$ and $x = 5$, so there are 2 **(B)** total values.

10. The square must have side length $\sqrt{36} = 6$. Thus, the length of the rectangle on the left must be $\frac{12}{6} = 2$, so the side length of the square must be $6 - 2 = 4$. Then the square has area 16, so the area of the last rectangle is $36 - 12 - 16 = 8$ **(A)**.

11. The answer is all odd n. This is because the sum of n consecutive numbers is n multiplied by the median of these numbers. For this to be a multiple of n, the median of these numbers must be an integer, which occurs for **odd** n **(C)**.

12. Note that $10^x - 100 = 100 \cdot (10^{x-2} - 1)$, which will have the form $99 \cdots 900$, where there are $x - 2$ nines. The two zeroes do not change the sum of the digits, so we have $(x - 2) \cdot 9 = 45$. Solving gives $n = 7$ **(D)**.

13. Ella can take any coin from any column, so she has $3 \cdot 2 \cdot 1 = 6$ choices of three coins to take.

    Once Ella takes the coins, there will be two coins in one column and one coin in the other, so Bella has $2 \cdot 1 = 2$ choices of coins to take.

    Cassandra is forced to take the only coin that is left, so there are $6 \cdot 2 \cdot 1 = 12$ **(B)** choices for the three of them.

14. If the first flip lands heads, the second flip has 1 choice as it must land tails. Then, the third and fourth flips can be either heads or tails because in any case the passerby still flips all four coins, so there are $1 \cdot 2 \cdot 2 = 4$ combinations for this case.

    If the first flip lands tails and the second flip lands heads, the third flip must land tails. Then the fourth flip can be either heads or tails, so there are $1 \cdot 2 = 2$ combinations for this case.

    If the first two flips land tails, the third and fourth flips can be either heads or tails. Then this case has $2 \cdot 2 = 4$ combinations for this case.

    Summing gives $4 + 2 + 4 = 10$ **(D)**.

15. We want to minimize the integer part of the mixed number, or a.

    If $a = 0$, we clearly must have $\frac{b}{c} = \frac{2}{9}$ as the only solution; then $x = \frac{2}{9}$.

    If $a = 1$, we have $\frac{1+b}{c} = \frac{2}{9}$. Hence $9 + 9b = 2c$, so c is a multiple of 9.

    Now note that $\frac{b}{c} = \frac{2}{9} - \frac{1}{c}$, so we want to minimize the value of c to minimize $\frac{b}{c}$.

    Substituting $c = 9$ gives $b = 1$, so the second value is $x = 1\frac{1}{9} = \frac{10}{9}$.

    Substituting $c = 18$ gives $b = 3$, which is not simplified, so we must move on to $c = 27$. This gives $b = 5$, which works, so the last value is $x = 1\frac{5}{27} = \frac{32}{27}$.

    The sum of these three values is $\frac{2}{9} + \frac{10}{9} + \frac{32}{27} = \frac{68}{27}$ **(B)**.

16. Let M be the midpoint of AB, and let x be the side length of $\triangle ABC$.

    Then we have $MP = BP - BM = 6 - \frac{x}{2}$, and $MC = \sqrt{x^2 - \left(\frac{x}{2}\right)^2} = \frac{x\sqrt{3}}{2}$.

    Now, Pythagorean Theorem on right triangle CMP gives $\left(6 - \frac{x}{2}\right)^2 + \left(\frac{x\sqrt{3}}{2}\right)^2 = 9^2$.

    This simplifies to $\frac{x^2}{4} - 6x + 36 + \frac{3x^2}{4} = 81$, or $x^2 - 6x - 45 = 0$. Then, solving for x using the Quadratic Formula gives $x = \frac{6+6\sqrt{6}}{2} = 3 + 3\sqrt{6}$ **(D)**

17. Note that after every two moves, Jane and Jena will always be opposite each other again. This is because after Jane moves, the two are next to each other, so Jena can only move to the other adjacent spot, which is opposite Jane.

    Every two moves, Jane will have 2 choices of chairs to move to, and Jena will only have 1, so there are 2 ways total to make each set of two moves.

    There are four of these sets of two moves and one final move for Jane, which gives $2^4 \cdot 2 = 32$ **(C)**.

18. Call the six-digit number N. In order for N to be divisible by 9, the sum of digits of N must be divisible by 9. Then we have that 9 divides $2 + 0 + 2 + 1 + a + b = 5 + a + b$, so either $a + b = 5$ or $a + b = 13$. The greatest possible value of $a \cdot b$, then, occurs at $a = 6$ and $b = 7$ (or $a = 7$ and $b = 6$), which corresponds to the answer choice 42 **(E)**.

19. Since N is a multiple of 36, it must also be a multiple of 9. This means that the sum of the digits of N is also a multiple of 9.

    Thus N is equal to 36 times a multiple of 9, meaning that N is a multiple of 324.

    There are only three 3-digit multiples of 324: 324, 648, and 972. Quickly testing each one reveals that only the first two work, so their sum is $324 + 648 = 972$ **(C)**.

20. Let a be the length of the vertical side of the dark gray triangle. Then the area of the dark gray triangle is $a \cdot 14 \cdot \frac{1}{2} = 7a$. This is equal to 35, so $a = 5$.

    Now, let b denote the length of the vertical edge of the light gray triangle. From similar triangles, we know that $\frac{b}{24} = \frac{a}{10}$, so $b = \frac{24a}{10} = 12$. The length of the height to this side is 24, so the area of the light gray triangle is $\frac{1}{2} \cdot 12 \cdot 24 = 144$ **(D)**.

21. Note that none of the answer choices are hypotenuses, so they must be legs of a right triangle. Let $x > y$ be the other side lengths of the right triangle, and a be the given length. We have $x^2 = y^2 + a^2$ by the Pythagorean Theorem and $a^2 = x^2 - y^2 = (x - y)(x + y)$. Observe that the pair $(x - y, x + y)$ directly correspond to the factors of a^2 and the number of such pairs increases if a has a higher power of 2, as $x - y$ and $x + y$ cannot be of opposite parities. The answer choice 24 **(C)** has the highest power of 2 among all five answer choices.

22. Let J be Jerry's number and A be Aaron's number.

    Since we know that Aaron's number is at least three times Jerry's number, the possible ordered pairs (J, A) are limited to $(1, 3)$, $(1, 4)$, $(1, 5)$, $(1, 6)$, and $(2, 6)$.

    Consider the first statement. If Aaron does not know Jerry's number, then Aaron must have the number 6, because otherwise he would have been able to determine that Jerry's number could only have been 1.

    Now consider the second statement. This statement implies that prior to Aaron's announcement, Jerry did not know Aaron's number. If Jerry had the number 2, he would have known from the beginning what Aaron's number was, so Jerry must have the number 1.

    The sum of their numbers is $1 + 6 = 7$ **(D)**.

23. Note that the cells labeled 11, 8, 7, and 4 share no rows or columns, so $S = 11 + 8 + 7 + 4 = 30$. Using this, we can fill out whatever we can using plain arithmetic:

?	8		
10		7	8
11		8	9
6		3	4

    Now, it is impossible for us to directly find the values in any more cells, so we turn to algebra.

    For ease of wording, call the columns A, B, C, and D from left to right, and the rows 1, 2, 3, and 4; for example, the value in cell $C4$ is 3.

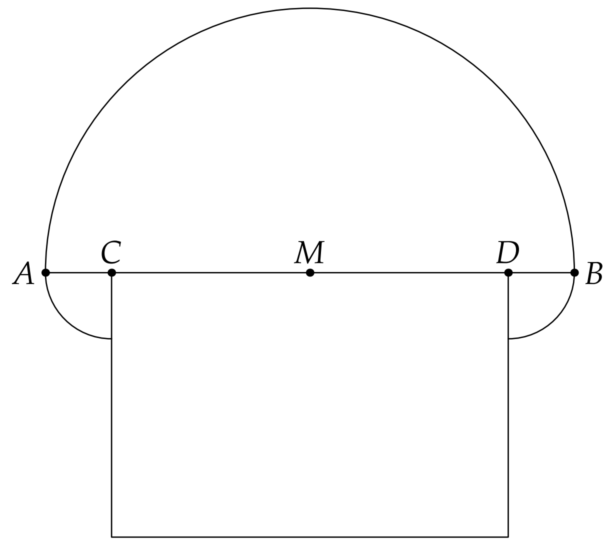
    Let the number in the cell $A1$ equal x. Then, from the diagonal extending from the top left to bottom right cells, cell $B2$ has a value of $18 - x$.

    Then, the third number in the top row has a value of $x - 3$, which is found by using cells $A3$, $B2$, $C1$, and $D4$.

    The last number in the top row will have a value of $x - 2$, which is found by using cells $A4$, $B2$, $C1$, and $D3$.

    Then we have $x + 8 + (x - 3) + (x - 2) = 30$. Solving gives $x = 9$ **(C)**.

24. The region that either cow can roam can be represented by a semicircle with radius 8 and two quarter circles, as shown below:



(The region is more or less the same on the other edge, though the radius of the quarter-circles vary).

The area of this region is the area of the semicircle plus twice the area of one of the quarter-circles; if the quarter circle has radius r, this is equal to $\left(\frac{1}{2}8^2 + \frac{1}{4} \cdot 2r^2\right)\pi = \left(32 + \frac{r^2}{2}\right)\pi$.

We set this equal to 34π and 40π to get $r = 2$ and $r = 4$, respectively. The side length CD must be equal to $AB - AC - BD = AB - 2AC = 16 - 2r$. Thus the two different side lengths are $16 - 2 \cdot 2 = 12$ and $16 - 2 \cdot 4 = 8$. The area of the rectangle is 96 **(C)**.

25. Since $2021 = 43 \cdot 47$ has $(1+1)(1+1) = 4$ divisors, $2021a$ must have $4 + 12 = 16$ divisors. Now it is fairly easy to simply test the answer choices:

If $a = 9$, then $2021a = 3^2 \cdot 43 \cdot 47$ which has $(2+1)(1+1)(1+1) = 12$ divisors, which fails, so choice A is incorrect.

If $a = 10$, then $2021a = 2 \cdot 5 \cdot 43 \cdot 47$, which indeed has $(1+1)(1+1)(1+1)(1+1) = 16$ divisors. This is what we wanted, so the answer is $a = 10$ **(B)**.